International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249-6890; ISSN (E): 2249-8001
Vol. 7, Issue 6, Dec 2017, 517-524

ON A DELAYED DIFFERENTIATION MULTI-PRODUCT FPR MODEL WITH SCRAP AND A MULTI-DELIVERY POLICY – II, USING TWO-MACHINE PRODUCTION SCHEME

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ABSTRACT

A finite production rate (FPR) system with delayed differentiation, scrap, and multi-delivery policy using a two-machine production scheme was explored by a conventional calculus method [1]. This paper provides an unconventional approach to this specific problem and demonstrates that its solution procedure can be free of derivatives. The production planner now has an alternative way to study such a complex FPR system.

KEYWORDS: Optimization, Finite Production Rate, Multi-Product System, Delayed Differentiation, Two-Machine Scheme, Common Cycle Time, Scrap and Multi-Delivery

Received: Oct 27, 2017; **Accepted**: Nov 17, 2017; **Published**: Dec 01, 2017; **Paper Id.**: IJMPERDDEC201759

Original Article

1. INTRODUCTION

In planning fabrication of multiple end products that share a common intermediate component, delayed differentiation is always a potential operational strategy aiming at reducing fabrication cycle time and/or relevant cost [1-9]. Gerchak et al. [2] developed a model, to explore multiple end products with joint demand distribution, common components, and simple form of cost structure. Gupta and Benjaafar [4] examined benefits of delaying differentiation in production systems, with variable order lead times. Effects of delayed differentiation on both make-to-stock and make-to-order types of systems, with different assumptions are investigated separately. Al-Salim and Choobineh [5], proposed two nonlinear models to explore the optimal differentiating timing, for product. Model one focused on profit maximization and model two was concerned with maximum options to postpone product differentiation. A tabu-constrained algorithm was employed to search for solutions to the models and parametric analysis was used to verify their results. Chiu et al. [1] employed the mathematical modeling along with optimization method to explore a delayed differentiation multi-product FPR model with scrap and discontinuous multi-shipment plan. A fabrication scheme comprising two-stage with two- machine was proposed in their study with the intention of shortening cycle time and reducing overall

Fabrication relevant costs. As a result, they not only determined closed-form optimal cycle time and number of shipments for the problem, but also achieved expected goals and revealed quite a few merits of this particular FPR system as compared to prior works [9-10]. Studies relating to fabrication systems with various aspects of quality improvements, multi-delivery plan, and rotation cycle time issues can also be referred to [11-20].

Grubbstrom and Erdem [21] presented a simplified approach, to solve the economic order quantity model with backordering without using derivatives. Studies applied the same or similar methods, to problems relating to various aspects of production lot sizing and vendor-buyer integrated inventory control can also be found in [22-23]. This paper uses such a straightforward methodology as well, to resolve a specific FPR problem studied in [1] and demonstrate that its solution procedure can be free of derivatives.

2. AN ALTERNATIVE APPROACH TO THIS SPECIFIC FPR PROBLEM

The particular multi-item FPR model explored by Chiu et al. [1] has the following features: a two-stage two-machine fabrication scheme with delayed differentiation, random scrap rate, and discontinuous product distribution plan. In stage one, machine one produces common parts for all products (Fig. 1) and in stage two, machine two fabricates finished products under a common production cycle time plan (Fig. 2). To ease readers' comparison efforts, the same nomenclature as in [1] is adopted and listed as follows:

$P_{1,0}$ = production rate of common parts

$t_{1,0}$= production time of common parts

$t_{2,0}$= production downtime of common parts

$t_{1,i}$= production time of finished products

$t_{2,i}$= production downtime of product i, when fixed quantity n installments of the finished batch are distributed, at a fixed interval of time, where i =1, 2… L,

Q_i=lot size of end product i in a production cycle

T=common production cycle time, one of the decision variables,

$P_{1,i}$ = production rate of end product i, where i =1, 2, …, L,

λ_i = annual demand rate of product i, where i =0, 1, 2, …, L,

Xi= percentage of scrap product i,

$d_{1,i}$ = production rate of scrap items of product i,

α =completion rate of common part,

C_i = unit production cost of product i,

$C_{S,i}$= unit disposal cost of product i,

K_i = setup cost of product i,

$h_{1,i}$ = unit holding cost of product i,

$h_{3,i}$=unit holding cost for stocks at buyer's side,

$h_{4,i}$=unit holding cost for safety stocks at manufacturer's side,

n =number of fixed quantity installments of the finished batch,

$t_{n,i}$=a fixed interval of time between each shipment of product i,

$K_{1,i}$= fixed shipping cost per delivery of product *i*,

CT,i= unit shipping cost of product i,

Hi=inventory level of common parts during fabrication time of product i,

H1, i=maximal level of finished product i in the end of production,

I (t) i=on-hand inventory level of perfect quality product i at time t, where i = 0, 1, 2, …L,

Id (t) i =on-hand inventory level of defective product i at time t,

Ic(t)i =on-hand inventory level of finished product i at time t, at customer's side,

E [TCU2 (T, n)] = the expected system cost per unit time for stage two,

E [TCU (T, n)] = the expected system per unit time for the particular FPR system.

As a result of mathematical modeling, derivation of system formulas, and cost analysis, the expected system cost per unit time, E [$TCU_2(T, n)$] can be found [1] as follows:

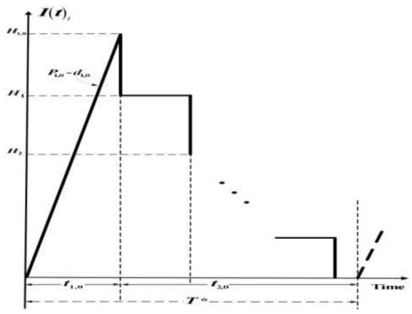

Figure 1: Inventory Status of Common Parts in the Stage one of the Proposed FPR System [1]

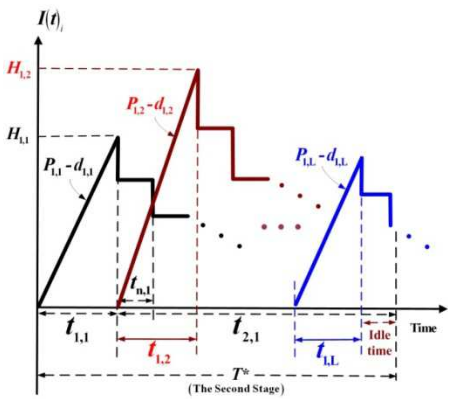

Figure 2: Inventory Status of Finished Products in Stage Two of the Proposed FPR System [1]

$$E\left[TCU_2\left(T,\ n\right)\right]=\sum_{i=1}^{L}\left\{\begin{array}{l}\left[C_i\lambda_iE_{0i}+\dfrac{K_i}{T}+C_{s,i}\lambda_iE_{1i}+\dfrac{nK_{1,i}}{T}+C_{T,i}\lambda_i\right]+Th_{4,i}\lambda_iE_{1i}\\+\dfrac{h_{1,i}T\lambda_i^2}{2}\left\{\left(1+\dfrac{1}{n}\right)\dfrac{E_{0i}^2}{P_{1,i}}+\left(1-\dfrac{1}{n}\right)\left[\dfrac{1-E\left[x_i\right]}{\lambda_i}E_{0i}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\}\\+\dfrac{h_{3,i}T\lambda_i^2}{2}\left\{\dfrac{2E_{0i}}{P_{1,i}}-\dfrac{1}{\lambda_i}+\left(1+\dfrac{1}{n}\right)\left[\dfrac{1-E\left[x_i\right]}{\lambda_i}E_{0i}-\dfrac{E_{0i}^2}{P_{1,i}}+\dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\}\end{array}\right\} \quad (1)$$

Where

$$E_{0i} = \frac{1}{\left(1 - E[x_i]\right)} \text{ and } E_{1i} = \frac{E[x_i]}{\left(1 - E[x_i]\right)}$$

Step 1: Deciding Optimal Number of Shipments for each Product I

It is noted that decision variables in E [$TCU_2(T, n)$] are of forms of T^{1}, T, Tn^{-1}, and nT^{1}. Let δ_a, δ_b, δ_c, δ_d, and δ_e represents the following coefficients:

$$\delta_a = \sum_{i=1}^{L}\left(C_i\lambda_i E_{0i} + C_{s,i}\lambda_i E_{1i} + C_{T,i}\lambda_i\right) \tag{2}$$

$$\delta_b = \sum_{i=1}^{L}\left(K_i\right); \ \ \delta_c = \sum_{i=1}^{L}\left(nK_{1,i}\right) \tag{3}$$

$$\delta_d = \sum_{i=1}^{L}\left\{\begin{array}{l} h_{4,i}\lambda_i E_{1i} + \dfrac{h_{1,i}\lambda_i^2}{2}\left\{\dfrac{E_{0i}^2}{P_{1,i}} + \left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i} + \dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} \\ + \dfrac{h_{3,i}\lambda_i^2}{2}\left\{\dfrac{2E_{0i}}{P_{1,i}} - \dfrac{1}{\lambda_i} + \left[\dfrac{1-E[x_i]}{\lambda_i}E_{0i} - \dfrac{E_{0i}^2}{P_{1,i}} + \dfrac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} \end{array}\right\} \tag{4}$$

$$\delta_e = \sum_{i=1}^{L}\left\{+\frac{h_{1,i}\lambda_i^2}{2}\left\{\frac{E_{0i}^2}{P_{1,i}} - \left[\frac{1-E[x_i]}{\lambda_i}E_{0i} + \frac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} + \frac{h_{3,i}\lambda_i^2}{2}\left[\frac{1-E[x_i]}{\lambda_i}E_{0i} - \frac{E_{0i}^2}{P_{1,i}} + \frac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\} \tag{5}$$

With these notations, Equation. (1) can be rearranged as follows:

$$E\left[TCU_2\left(T,\ n\right)\right] = \delta_a + \delta_b T^{-1} + \delta_c n T^{-1} + \delta_d T + \delta_e T n^{-1} \tag{6}$$

With additional rearrangements, Eq. (6) turns into the following:

$$E\left[TCU_2\left(T,\ n\right)\right] = \delta_a + \left(\sqrt{\delta_b} - \sqrt{\delta_d}T\right)^2 T^{-1} + \left(\sqrt{\delta_c} - \sqrt{\delta_e}Tn^{-1}\right)^2 nT^{-1} + 2\sqrt{\delta_b\delta_d} + 2\sqrt{\delta_c\delta_e} \tag{7}$$

It can be seen that if the second and third terms of Equation. (7) equal to zeros, then E[$TCU_2(T, n)$] can be minimized. That is

$$T = \sqrt{\frac{\delta_b}{\delta_d}} \text{ and } n = T\sqrt{\frac{\delta_e}{\delta_c}} = \sqrt{\frac{\delta_b\delta_e}{\delta_c\delta_d}} \tag{8}$$

Substitute δ_b, δ_c, δ_d, and δ_e in Equation. (8), n becomes

$$n=\sqrt{\frac{\sum_{i=1}^{L}[K_i]\cdot\sum_{i=1}^{L}\left\{\frac{\lambda_i^2}{2}\left(h_{3,i}-h_{1,i}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i}+\frac{E_{0i}E_{1i}}{P_{1,i}}-\frac{E_{0i}^2}{P_{1,i}}\right]\right\}}{\left(\sum_{i=1}^{L}\left(K_{1,i}\right)\right)\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{h_{1,i}\lambda_i^2}{2}\left\{\frac{E_{0i}^2}{P_{1,i}}+\frac{1-E[x_i]}{\lambda_i}E_{0i}+\frac{E_{0i}E_{1i}}{P_{1,i}}\right\}+h_{4,i}\lambda_iE_{1i}\\+\frac{h_{3,i}\lambda_i^2}{2}\left\{\frac{2E_{0i}}{P_{1,i}}-\frac{1}{\lambda_i}+\frac{1-E[x_i]}{\lambda_i}E_{0i}-\frac{E_{0i}^2}{P_{1,i}}+\frac{E_{0i}E_{1i}}{P_{1,i}}\right\}\end{array}\right\}}} \tag{9}$$

In real applications, n can only be integer, so let n^+ denote the smallest integer greater than or equal to n (i.e., from result of Eq. (9)), also let n^- be the largest integer less than or equal to n. Therefore, the optimal n^* is either n^+ or n^-.

Step 2: Determining the Optimal Cycle Time

Since n is obtained, now we treat E[$TCU_2(T, n)$] as a function with single decision variable and rearrange Equation. (6) as follows:

$$E\left[TCU_2\left(T,\ n\right)\right]=\delta_a+\left(\delta_b+\delta_c n\right)T^{-1}+\left(\delta_d+\delta_e n^{-1}\right)T \tag{10}$$

Or

$$E\left[TCU_2(T,n)\right]=\delta_a+T^{-1}\left(\sqrt{\delta_b+\delta_c n}-T\sqrt{\delta_d+\delta_e n^{-1}}\right)^2+2\sqrt{\delta_a+\delta_b n}\sqrt{\delta_c+\delta_d n^{-1}} \tag{11}$$

It can be seen that if the second square term of Equation. (11) equals zero, then E[$TCU_2(T, n)$] can be minimized,. That is

$$T^*=\sqrt{\frac{\delta_b+\delta_c n}{\delta_d+\delta_e n^{-1}}} \tag{12}$$

Substitute δ_b, δ_c, δ_d, and δ_e in Equation. (12), T^* is found as follows:

$$T^*=\sqrt{\frac{\sum_{i=1}^{L}\left[K_i+nK_{1,i}\right]}{\sum_{i=1}^{L}\left\{\begin{array}{l}\frac{h_{1,i}\lambda_i^2}{2}\left\{\left(1+\frac{1}{n}\right)\left[\frac{1}{P_{1,i}}E_{0i}^2\right]+\left(1-\frac{1}{n}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i}+\frac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\}\\+\frac{h_{3,i}\lambda_i^2}{2}\left\{\frac{2E_{0i}}{P_{1,i}}-\frac{1}{\lambda_i}+\left(1+\frac{1}{n}\right)\left[\frac{1-E[x_i]}{\lambda_i}E_{0i}-\frac{E_{0i}^2}{P_{1,i}}+\frac{E_{0i}E_{1i}}{P_{1,i}}\right]\right\}+h_{4,i}\lambda_iE_{1i}\end{array}\right\}}} \tag{13}$$

Equations (9) and (13) are the same as that was found in [1]. It follows that to obtain the expected system cost per unit time E[$TCU(T, n)$] for the proposed FPR system, same solution process in [1] can be employed (please refer to Esq. (26) and (27) in [1]).

3. CONCLUSIONS

An unconventional approach is presented in this study to resolve a particular FPR problem considering, random scrap, multi-delivery plan, and delayed differentiation using a two-machine fabrication scheme. Unlike prior study [1]

employed conventional differential calculus method, the proposed simplified solution process is free of derivatives and it offers production planners with an alternative way of investigation of such a complex FPR system. According to obtained results, sensitivity analysis, performance, and benefits of this particular FPR system can also be gained as that in [1].

ACKNOWLEDGEMENTS

Authors deeply appreciate Ministry of Science and Technology, ROC (Taiwan) for sponsoring this study under grants#: MOST 102-2410-H-324-015-MY2 and and MOST 104-2221-E-324-029-MY3.

REFERENCES

1. *Chiu, S.W., Hsieh, Y-T., Chiu, Y-S.P., Hwang, M-H. 2016. A delayed differentiation multi-product FPR model with scrap and a multi-delivery policy – II: Using two-machine production scheme. International Journal for Engineering Modelling 29(1-4): 53-68.*
2. *Gerchak, Y., Magazine, M.Y., Gamble, B.A. 1988. Component commonality with service level requirements. Management Science 34(6): 753-760.*
3. *Swaminathan, J.M., Tayur, S.R. 1998. Managing broader product lines through delayed differentiation using vanilla boxes. Management Science 44(12 Part 2): S161-S172.*
4. *Gupta, D., Benjaafar, S. 2004. Make-to-order, make-to-stock, or delay product differentiation? A common framework for modeling and analysis. IIE Transactions 36(6): 529-546.*
5. *Al-Salim, B., Choobineh, F. 2009. Redesign of production flow lines to postpone product differentiation. International Journal of Production Research 47(20): 5563-5590.*
6. *Graman, G.A. 2010. A partial-postponement decision cost model. European Journal of Operational Research 201(1): 34-44.*
7. *Bernstein, F., DeCroix, G.A., Wang, Y. 2011. The impact of demand aggregation through delayed component allocation in an assemble-to-order system. Management Science 57(6): 1154-1171.*
8. *Lu, C-C., Tsai, K-M., Chen, J-H. 2012. Evaluation of manufacturing system redesign with multiple points of product differentiation. International Journal of Production Research 50(24): 7167-7180.*
9. *Chiu, Y-S.P., Huang, C-C., Wu, M-F., Chang, H-H. 2013. Joint determination of rotation cycle time and number of shipments for a multi-item EPQ model with random defective rate. Economic Modelling 35: 112-117.*
10. *Chiu, Y-S.P., Hsieh, Y-T., Kuo, J-S., Chiu, S.W. 2016. A delayed differentiation multi-product FPR model with scrap and a multi-delivery policy – I: using single-machine production scheme. International Journal for Engineering Modelling 29(1-4): 37-52.*
11. *Masoumik, S.M., Abdul-Rashid, S.H., Olugu, E.U. 2015. The development of a strategic prioritisation method for green supply chain initiatives. PLoS ONE 10(11): art. no. e0143115.*
12. *Fergany, H.A. 2016. Probabilistic multi-item inventory model with varying mixture shortage cost under restrictions. SpringerPlus 5(1): art. no. 1351.*
13. *Chiu, Y-S.P., Chiang, K-W., Chiu, S.W., Song, M-S. 2016. Simultaneous determination of production and shipment decisions for a multi-product inventory system with a rework process. Advances in Production Engineering & Management 11(2): 141-151.*
14. *Dinesh, K.S., Ashwanth, A.R., Karthikeyan, S., Nithyanandam, G.K. 2016. Investigation to improve the new product development process of mono block pump using additive manufacturing. International Journal of Mechanical and Production*

Engineering Research and Development 6(2), 77-88.

15. *Balaji, M., Velmurugan, V., Subashree, C. 2015. TADS: An assessment methodology for agile supply chains. Journal of Applied Research and Technology 13(5):504-509.*

16. *Chiu, Y-S.P., Liang, G-M., Chiu, S.W. 2016. Solving a fabrication lot-size and shipping frequency problem with an outsourcing policy and random scrap. Mathematical and Computational Applications 21(4): Art. No. 45.*

17. *Aboumasoudi, A.S., Mirzamohammadi, S., Makui, A. & Tamosaitiene, J. 2016. Development of network-ranking model to create the best production line value chain: A case study in textile industry. Economic Computation and Economic Cybernetics Studies and Research 50(1): 215-234.*

18. *Bazdar, A., Kazemzadeh, R.B., Niaki, S.T.A. 2015. Variation source identification of multistage manufacturing processes through discriminant analysis and stream of variation methodology: A case study in automotive industry. Journal of Engineering Research 3(2): 96-108.*

19. *Kaylani, H., Almuhtady, A., Atieh, A.M. 2016. Novel approach to enhance the performance of production systems using lean tools. Jordan Journal of Mechanical and Industrial Engineering 10(3): 215-229.*

20. *Zahedi, Z., Ari Samadhi, T.M.A., Suprayogi, S., Halim, A.H. 2016. Integrated batch production and maintenance scheduling for multiple items processed on a deteriorating machine to minimize total production and maintenance costs with due date constraint. International Journal of Industrial Engineering Computations 7(2): 229-244.*

21. *Grubbstrom, R.W., Erdem, A. 1999. The EOQ with backlogging derived without derivatives. International Journal of Production Economics 59: 529-530.*

22. *Lin, H-D., Wu, M-F., Chiu, Y-S.P. 2017. On mathematical modeling of a multi-product EMQ model with an enhanced end items issuing policy and failures in rework. Far East Journal of Mathematical Sciences 102(3): 573-583.*

23. *Wu, M-F., Chiu, T., Lin, H-D., Chiu, S.W. 2017. On effect of delayed differentiation on a multiproduct vendor–buyer integrated inventory system with rework. International Journal of Applied Engineering Research 12(19): 8144-8149*